Rear Car Differential

ENGR 3350 Manufacturing Processes

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List of Nomenclature

b	Contact half-width
b_w	Operating tooth width (in.)
$b_{wF1,2}$	Operating tooth width (in.)
$C_{f1,2}$	Surface condition factor
$C_{H1,2}$	Hardness ratio factor
C_p	Pinion proportion factor
d_1	Diameter 1 (in.)
d_2	Diameter 2 (in.)
d_{w1}	Pitch diameter (in.)
E_1	Elastic modulus 1 (ksi)
E_2	Elastic modulus 2 (ksi)
F	Applied force (N)
F_t	Tangential Force (lb_f)
I	Contact length of cylinders (Hertzian contact analysis)
I	Geometry factor for pitting resistance (safety factor of contact fatigue)
$J_{1,2}$	Geometry factor for bending strength
$K_{B1,2}$	Rim thickness factor
$K_{m1,2}$	Load distribution factor
K_o	Overload factor
K_R	Reliability factor
$K_{S1,2}$	Stress cycle factor for pitting resistance
K_T	Temperature factor
K_v	Size factor
Pmax	Maximum pressure (PSI)
P_t	Tangential diametral pitch (in^{-1})
S_{ac}	Allowable Contact Stress (Pa)
$S_{at1,2}$	Allowable Bending Stress (Pa)
$S_{F1,2}$	Safety factor of bending fatigue

$S_{H1,2}$ Safety factor of contact fatigue

v_1 Poisson's ratio 1

v_2 Poisson's ratio 2

$Y_{A1,2}$ Alternating factor

$Y_{N1,2}$ Stress cycle factor for bending strengths

Z Depth below the surface (in.)

$Z_{N1,2}$ Stress cycle factor for pitting resistance

π Pi

σ_x Principal stress

σ_y Principal stress

σ_z Principal stress

τ_{xz} Shear stress

τ_{yz} Shear stress

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Abstract

This lab focused on designing and building a small-scale functional model of a rear car differential to understand its mechanical operation and real-world application. The model was constructed using aluminum shafts for durability, 3D-printed gears for customizable design and lightweight properties, and a plywood case for accessible and sturdy housing. This scaled-down version mimics the functionality of a full-sized differential, which is essential in vehicles for distributing torque and allowing wheels to rotate at different speeds during turns. The report outlines the steps taken in the design, fabrication, and assembly process, as well as an analysis of the model's performance and key observations.

Theory

The rear car differential is a critical component in automotive design that allows the drive wheels to spin at different speeds. This functionality is achieved by a set of spider gears which can immediately adjust the speed of the wheels just by the restive forces experienced in the axles. When designing the gear sets for the differential a few parameters had to be named. Specifically, how fast the assembly would spin, and the maximum power input to the system. With this the team picked arbitrary values, The maximum power input was decided to be 0.1 horsepower, and the rotational speed was decided to be 100RPM. Using the Inventor design accelerator, the team was able to input all necessary design parameters for all gear sets allowing inventor to generate all gear sets. The team then performed all the inventor calculations using ANSI/AGMA 2001-D04 to calculate all possible values that could be used in the design process. The team then performed Hertzian contact calculations to evaluate the maximum shear stress that the designed teeth would experience to further determine whether the design was adequate. After the calculations it was confirmed that the designed gears would work. The team carried on with the design process going through a multitude of iterations. Once a design was settled upon a Von-mises stress analysis was performed. The stress analysis would further confirm the structural soundness of the assembly (Figure 1). The team set multiple design goals. These beings make a working differential, create 3 shafts out of aluminum instead of 3D printing them, and finally make the case out of plywood to limit the costs.

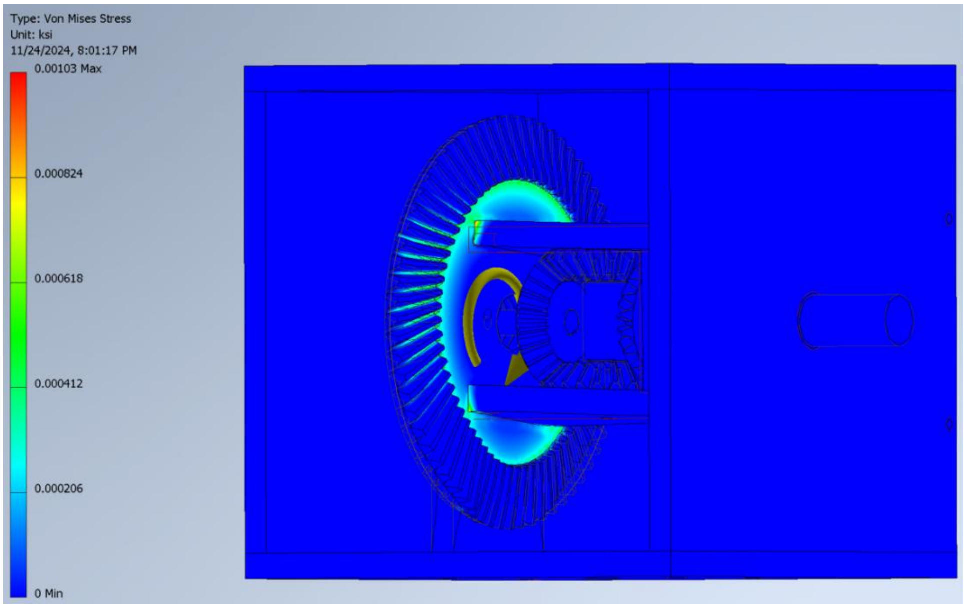


Figure 1. Stress Analysis of Final Assembly.

Equipment

1. 3D Printer (Stratasys F170)
2. Band Saw (Metal Mizer 2018)
3. Belt Sander-Grinder
4. Double Column Height Dial Gauge
5. Lathe
6. Milling Machine
7. Table Saw

Procedure

Inventor

<u>Ring Gear and Pinion Gear</u>

1. Open the Inventor design accelerator and select bevel gears.
2. Input your desired parameters.
3. Modify your material properties for the appropriate material.
4. Check that your parameters will produce adequate gears by using the calculations tab.
5. In the calculations tab change the standard to ANSI/AGMA 2001-D04.

6. Make any necessary modifications to the generated gears such as adding holes (Figures 2, 3).
7. Save as a .STL file
8. Use the 3D printing software to print the parts at 50% infill
9. Perform any necessary finishing work will depend on the quality of print

Spider Gears

1. Open the Inventor design accelerator and select bevel gears.
2. Input your desired parameters.
3. Modify your material properties for the appropriate material.
4. Check that your parameters will produce adequate gears by using the calculations tab.
5. In the calculations tab change the standard to ANSI/AGMA 2001-D04.
6. Make any necessary modifications to the generated gears such as adding holes or keys (Figure 2, 3).
7. Save as a .STL file
8. Use the 3D printing software to print the parts at 50% infill
9. Perform any necessary finishing work will depend on the quality of print
10. Press fit the “Self-Aligning Dry-Running Flanged Sleeve Bearings” into the center hole of the non-keyed spider gear (Figure 4).

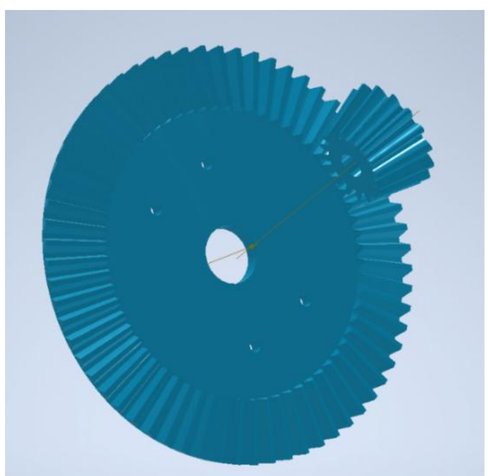


Figure 2. Ring and Pinion assembly

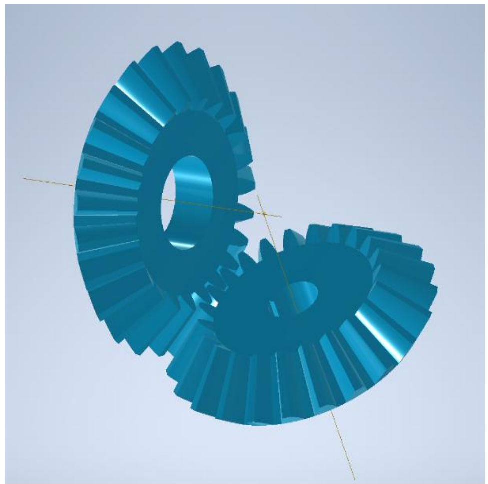


Figure 3. Spider Gear Assembly

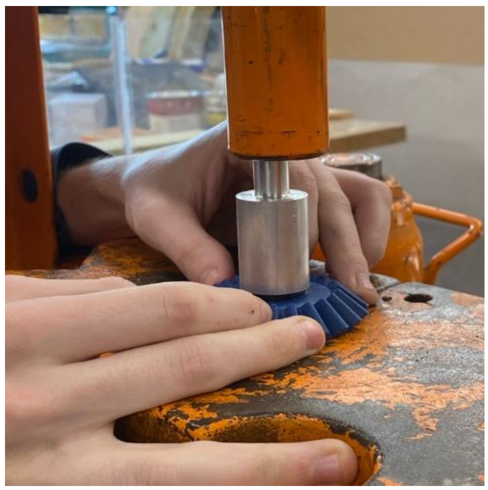


Figure 4. Press fitting the bearings into the gears

Journals and Journal Caps

1. Design a journal for the roller to roll in. The journal should be at least 0.500-inch larger than the roller diameter.
2. Design a cap that will enclose the roller and stop it from slipping out of the assembly.
3. Use the inventor threaded hole tool to create ¼-20 threads 1.250-inch deep to secure the cap to the journal and holes on the bottom of the journal.
4. Save as a .STL file.
5. Use the 3D printing software to print the parts.
6. Remove the support material from the print.
7. Test that the printed holes can fit the ¼-20 bolts.

Rollers

1. Create a cylinder that can accommodate two 0.250-inch wooden dowels with an inner diameter larger enough for a 0.500-inch shaft to pass through. The outer diameter must be large enough for 0.250-inch holes to fit between the outside and inside permitter of the cylinder (Figure 7)
2. Make the cylinder 0.500-inch long
3. Save as a .STL file
4. Use the 3D printing software to print the parts
5. Remove the support material from the print

Figure 5. Journal Cap

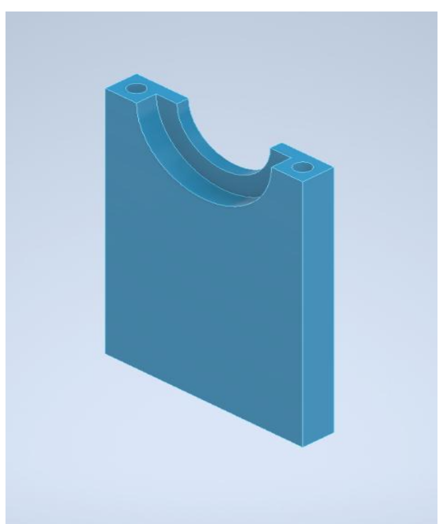


Figure 6. Journal

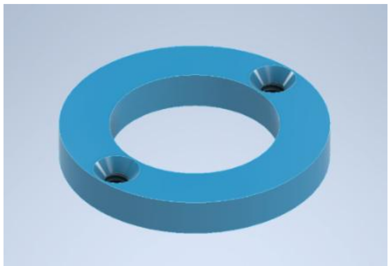


Figure 7. Roller

3D-Model

1. Create an assembly of all the parts in inventor.
2. Complete a Von-Mises Stress analysis on the assembly.
3. Analyze the stress analysis and determine if any modifications need to be made.

Shafts

Spider Gear Connecting shaft

1. Cut Aluminum round stock at 3 inches on band saw.
2. Reduce length to 2.742 inch. on lathe (Figure 8, 9).
3. Using a #7 drill bit, create a 0.500-inch. deep hole centered onto one face of the shaft (Figure 10)
4. Create a ¼-20 thread. Using a taper, plug, and bottom tap respectively (Figure 11).
5. Repeat steps 3 and 4 on the other side of the shaft.
6. Use double height column gage to mark at 0.609-inch from edge of shaft, this will be depth of reduced diameter.
7. Reduce diameter of shaft to 0.500-inch on lathe (Figure 12).
8. Chamfer edge of reduced diameter (Figure 13).
9. Repeat steps 6-8 to the other side of the shaft.

Figure 8. Reducing length of spider gear shaft

Figure 9. Final length of spider gear shaft

Figure 10. Creating center hole for threads

Figure 11. Threading spider gear shaft hole

Figure 12. Reducing diameter of spider gear shaft

Figure 13. Adding chamfer to reduced diameter

Pinion shaft

1. Cut Aluminum round stock to approximately 5.125-inch on the band saw
2. Reduce length to 5-inch on lathe (Figure 14).
3. Use double height column gage to mark at 2.300-inch from edge of shaft, this will be depth of reduced diameter.
4. Reduce diameter to 0.500-inch using the lathe (Figure 15)
5. Chamfer edge of reduced diameter (Figure 16).
6. Use double height column gage to measure 1.210-inch from edge to mark how long keyway will be (Figure 17).
7. Using 7/64-inch endmill to create keyway at depth of 0.055-inch (Figure 18) on the milling machine.

Figure 14. Final length of pinion shaft

Figure 15. Reducing diameter of pinion shaft

Figure 16. Adding chamfer to pinion shaft

Figure 17. Marking where keyway will end

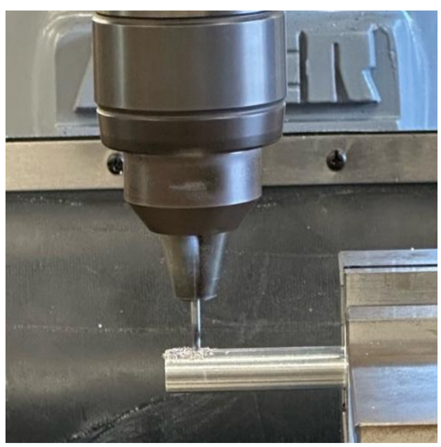


Figure 18. End milling keyway into pinion shaft

Spider gear side shaft

1. Cut Aluminum round stock to approximately 6-1/8 inches on band saw
2. Reduce length to 6 inches on lathe.

3. Use double height column gage to mark at 3.983 inches from edge of shaft, this will be length of reduced diameter.
4. Reduce diameter to 0.50 inches using the lathe (Figure 19).
5. Chamfer edge of reduced diameter (Figure 20).
6. Use double height column gage to measure 0.50 inches from edge to mark how long keyway will be.
7. Using 7/64-inch endmill to create keyway at depth of 0.055 inch using milling machine on reduced diameter (Figure 21).

Figure 19. Reducing diameter of spider gear side shaft

Figure 20. Adding chamfer to spider gear side shaft

Figure 21. Adding keyway on spider gear side shaft

Ring gear side shaft

1. Cut the Aluminum round stock to approximately 6.125-inch using the band saw (Figure 22).
2. Reduce length of shaft on lathe to 5.970-inch.
3. Use the double height column gage to mark at 4.330-inch from edge of shaft, this will be length of reduced diameter.
4. Reduce the diameter of shaft to 0.500-inch.
5. Use double height column gage to measure 0.500-inch from the edge of the reduced diameter side to mark how long keyway will be.
6. Using a 7/64-inch endmill to create the keyway at a depth of 0.055-inch using the milling machine on reduced diameter (Figure 23).
7. Completed ring gear side shaft (Figure 24).

Figure 22. Cutting shaft using bandsaw

Figure 23. Adding keyway on ring gear side shaft

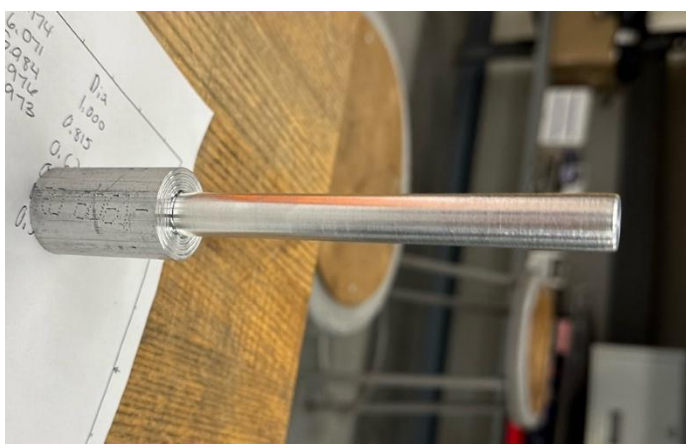


Figure 24. Ring gear side shaft

Carrier

Top Plate

1. Cut 0.500-inch plywood into a 2.750-inch by 3-inch rectangle using a table saw
2. Drill a 1.125-inch diameter hole though the center
3. Drill a 0.250-inch diameter hole 0.875-inch from center above and below the center hole. These holes are 1.500 inches away from the 2.75-inch-long sides of the part (Figure 25).

Side Plate

1. Cut 0.500-inch plywood into a 4.5 inch by 3-inch rectangle using a table saw
2. Drill a .25-inch diameter hole 2.25 inches from the 3-inch-long side, and 1.500-inch from the 4.5-inch side.
3. On the 3-inch by 0.5-inch side, using the height gage mark a location 0.25-inch from the top and another on the bottom side.
4. Drill a 0.250-inch hole on those marks using a smaller bit at least 7/64-inch in diameter to create a pilot hole. Then use a 0.250-inch drill bit to make the final hole (Figure 26).
5. Repeat steps 1 to 4 to create a second identical plate

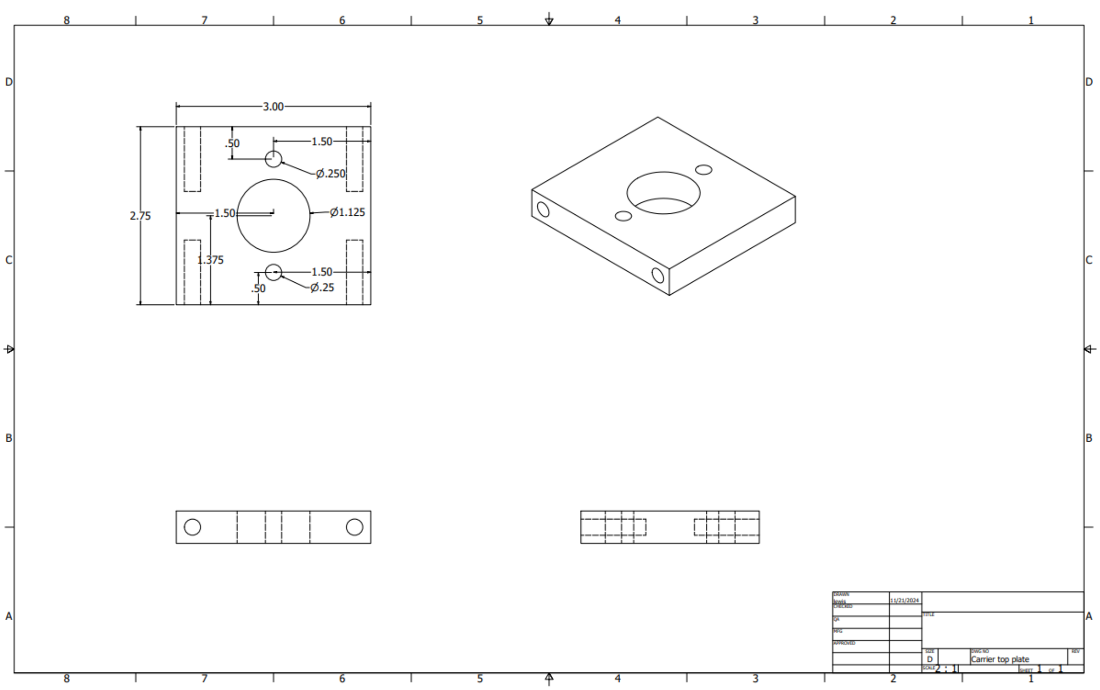


Figure 25. Top Plate Dimensions

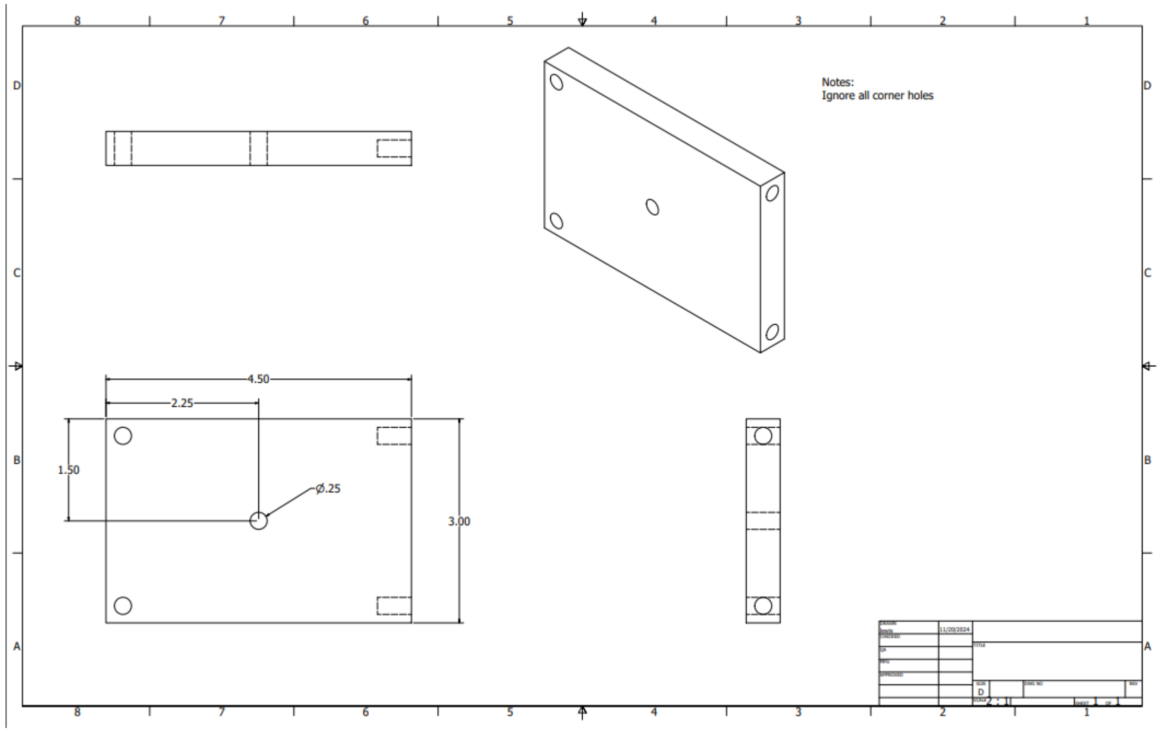


Figure 26. Side Plate Dimensions

Assembly

1. On both side plates, insert 0.25-inch wooden dowels covered in wood glue into the 0.25-inch holes. Use a mallet to tamp them in.
2. The dowels should be protruding from the plate, insert the protruding dowels into the appropriate holes on the ring gear. Do this for both plates
3. Once inserted if the dowels protrude through the ring gear, go to the sander and sand down the dowels until flush with the back face of the ring gear.
4. Take the 3D printed rollers and press dowels through the two holes.
5. Press the protruding dowels into the holes on the top plate. Again, if the dowels protrude sand them down until they are flush with the plate.
6. Insert the top plate in between the two side plates, the roller should be visible when looking at it from the top down.
7. Using finishing nails, hammer the top plate into place. Use at least 1 nail for each corner and be careful not to split the plywood.
8. Set aside and let the wood glue dry

9. Take the spider gear connecting shaft and the spider gears with the sleeve bearings and slide the gears onto the shaft. The teeth of the gears should be facing each other (Figure 27).
10. Slide this shaft into the center of the carrier aligning the bolt holes with the holes on the side plate.
11. Once inserted fasten the bolts into the carrier securing the spider gear shaft.
12. With the remaining roller, insert the dowels into the only remaining holes on the ring gear. The roller should be on the back face of the ring gear.
13. Check all dimensions on the assembled carrier.
14. If dimensions are not the expected size, make the necessary adjustments to meet design specifications. Whether that is shimming the plates or sanding any surfaces.
15. Sand the back fact of the ring gear until smooth.

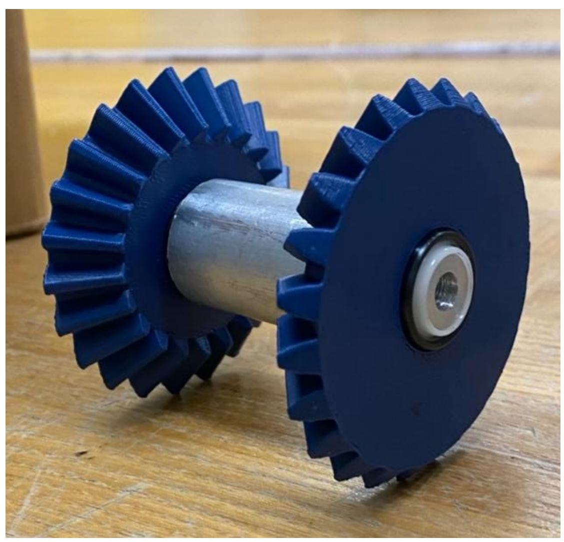


Figure 27. Assembled Spider Gear Shaft

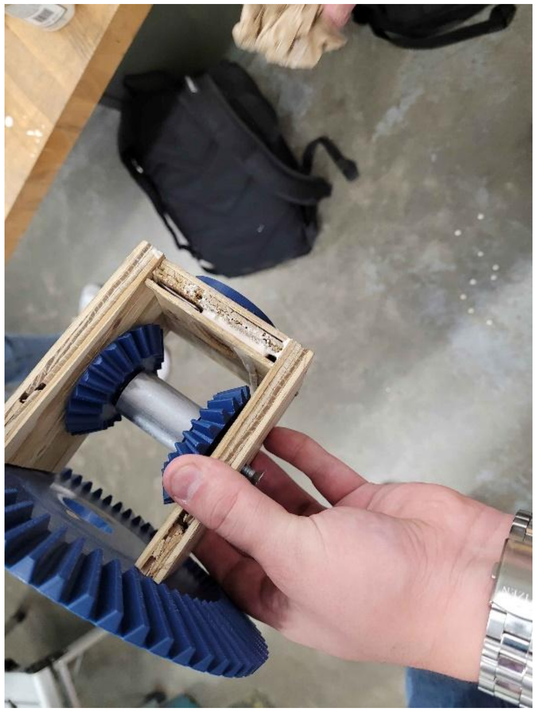


Figure 28. Assembled Carrier

Case

Back Plate

1. Cut 0.500-inch plywood into a 9-inch square using a table saw
2. Cut a 1.125-inch diameter hole 3.226-inch from one side, and 4.5inch from the adjacent side.

Outer Casing – Side Plate

1. Cut 0.500-inch plywood into a 9-inch by 10-inch rectangle using a table saw
2. Using the heigh gage mark lines from the middle of all sides.
3. Mark the intersection of these lines. That is the center of the plate
4. Drill a 1.125-inch diameter hole into the plate for a center hole
5. Repeat steps 1-4 to make a second side plate.

Outer Casing – Bottom Plate

1. Cut 0.500-inch plywood into a 10-inch by 10-inch plate using a table saw
2. Using the height gage measure 3.665-inch up from a side and make a mark across the plate. Rotate the plate 180 degrees and repeat

3. Rotate the plate to one of the adjacent sides and make a line 2.344 inch above the side. Again, rotate the plate 180 degrees and make the same mark.
4. Mark the intersections on the plate using a sharpie.
5. Using a 0.25-inch drill bit, drill a hole on each marked intersection through the plate.

Final Assembly

1. Starting with the bottom plate, fasten both journals to the bottom plate using the ¼-20 bolts.
2. Check that the journals are level to each other, and one is not taller than the other. If so tighten or loosen some of the bolts until level.
3. Rest the assembled carrier onto the journals the rollers should slot onto the journals.
4. Roll the carrier assembly to make sure there is no rubbing or friction. If there is friction sand down any areas that are causing interference.
5. Once rolling smooth fast the journal caps to secure the assembled carrier.
6. Using finished nails, nail the side plates to the left and right side of the carrier. Those are the sides with the rollers.
7. Test the concentric tolerances by running a shaft through the hole of the side plate into the carrier.
8. Nail the back plate onto the bottom plate it does not matter which side as long as the hole of the back plate aligns with the teeth on the ring gear.
9. Using wood glue, glue the plastic bearings into the holes of all three plates.
10. Once glue has dried, insert all shafts into their appropriate holes. Pinion shaft on the back plate. Ring gear shaft on the ring gear side, and the remaining shaft goes into the last hole.
11. Press the gears onto the shafts aligning the keys by hand and make sure the gears mesh. (Figure 29, 30)
12. Check and see if all shafts spin appropriately

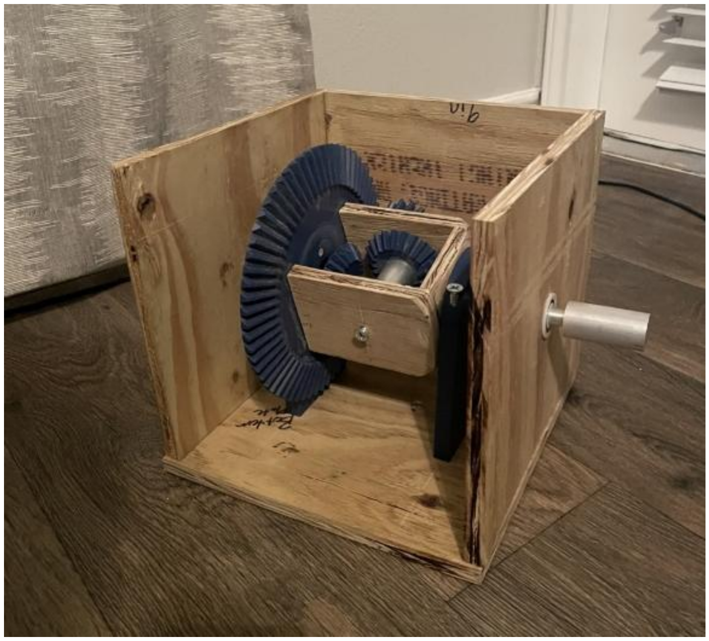


Figure 29. Assembled Differential

Figure 30. Top-down View of Assembled Differential

Data

<u>Place of experiment:</u> RFEB 114

Table 1: Length of Aluminum round stocks using calipers

	Initial length (inches)	Final length (inches)
Ring gear side shaft	6.174	5.973
Spider gear side shaft	6.121	6.007
Pinion shaft	5.197	5.000
Spider gear connecting shaft	2.864	2.742

Table 2: Diameter of Aluminum round stocks using calipers

	Initial length (inches)	Final length (inches)
Ring gear side shaft	1.000	0.504
Spider gear side shaft	1.005	0.499
Pinion shaft	1.003	0.507
Spider gear connecting shaft	1.000	0.49

Table 3: Design parameters for the ring and pinion gear

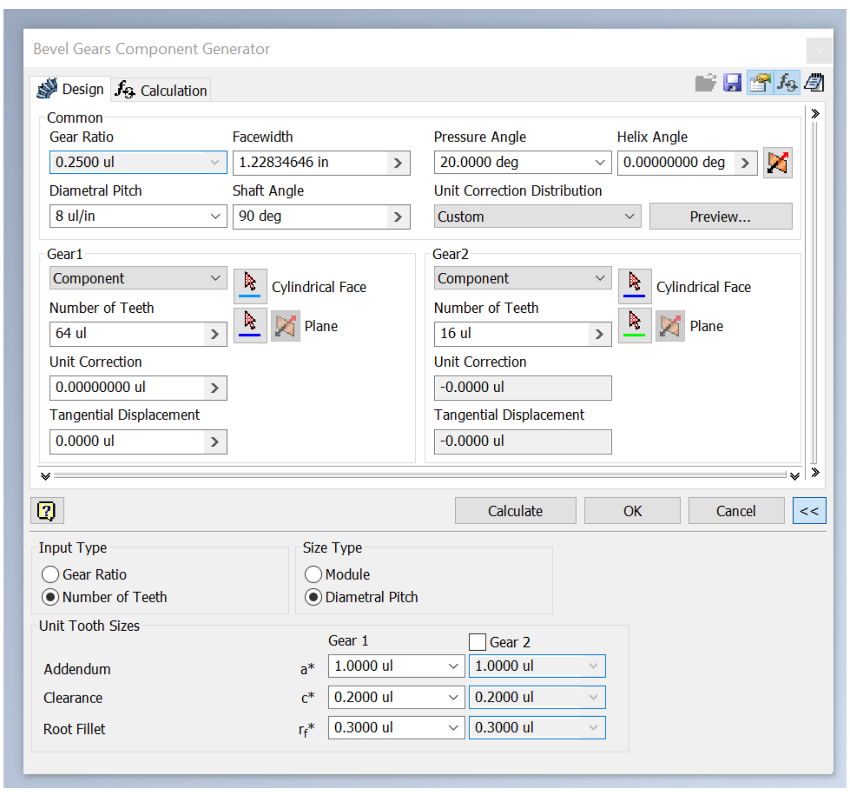


Table 4: Design parameters for the spider gears

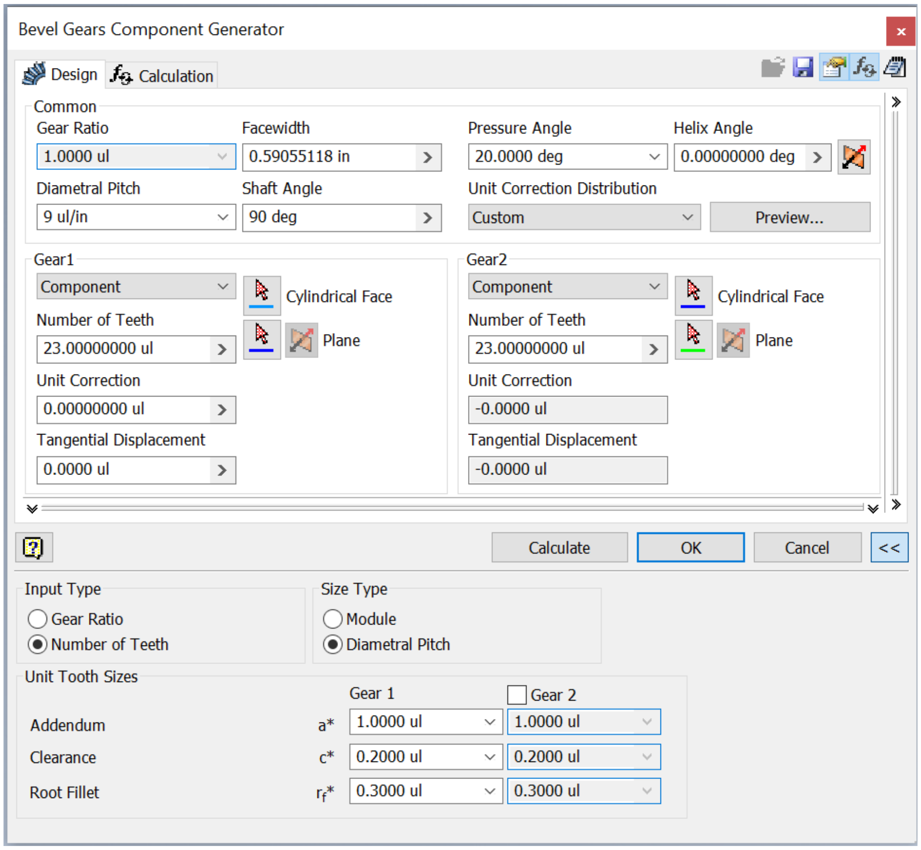


Table 5: Common parameters for the ring and pinion gear

⊟ Common Parameters

Gear Ratio	i	0.2500 ul
Tangential Diametral Pitch	P_{et}	8.000 ul/in
Helix Angle	β	0.0000 deg
Tangential Pressure Angle	α_t	20.0000 deg
Shaft Angle	Σ	90.0000 deg
Normal Pressure Angle at End	α_{ne}	20.0000 deg
Contact Ratio	ε	1.7109 ul
Limit Deviation of Axis Parallelity	f_x	0.00051 in
Limit Deviation of Axis Parallelity	f_y	0.00026 in
Virtual Gear Ratio	i_v	16.000 ul
Equivalent Center Distance	a_v	15.398 in
Virtual Center Distance	a_n	15.398 in
Pitch Cone Radius	R_e	4.123 in
Pitch Cone Radius in Middle Plane	R_m	3.623 in

Table 6: Loads experienced by the ring and pinion gear

⊟ Loads

		Gear 1	Gear 2
Power	P	0.100 hp	0.098 hp
Speed	n	100.00 rpm	400.00 rpm
Torque	T	5.252 lbforce ft	1.287 lbforce ft
Efficiency	η	0.980 ul	
Tangential Force	F_t	17.931 lbforce	
Normal Force	F_n	19.082 lbforce	
Radial Force (direction 1)	F_{r1}	1.583 lbforce	6.331 lbforce
Radial Force (direction 2)	F_{r2}	1.583 lbforce	6.331 lbforce
Axial Force (direction 1)	F_{a1}	6.331 lbforce	1.583 lbforce
Axial Force (direction 2)	F_{a2}	6.331 lbforce	1.583 lbforce
Circumferential Speed	v	3.067 fps	
Resonance Speed	n_{E1}	8532.753 rpm	

Table 7: Material properties of the ring and pinion gear

⊟ Material

		Gear 1	Gear 2
		User material	User material
Ultimate Tensile Strength	S_u	85300 psi	101500 psi
Yield Strength	S_y	45500 psi	49300 psi
Modulus of Elasticity	E	380000 psi	380000 psi
Poisson's Ratio	μ	0.400 ul	0.400 ul
Allowable Bending Stress	s_{at}	5500.0 psi	5500.0 psi
Allowable Contact Stress	s_{ac}	7500.0 psi	7500.0 psi
Hardness in Tooth Core	JHV	210 ul	210 ul
Type of Treatment	type	0 ul	2 ul

Table 8: Ring and pinion gear dimensions

⊟ Gears

		Gear 1	Gear 2
Type of model		Component	Component
Number of Teeth	z	64 ul	16 ul
Unit Correction	x	0.0000 ul	-0.0000 ul
Tangential Displacement	x_t	0.0000 ul	-0.0000 ul
Pitch Diameter at End	d_e	8.000 in	2.000 in
Pitch Diameter in Middle Plane	d_m	7.030 in	1.757 in
Outside Diameter at End	d_{ae}	8.061 in	2.240 in
Outside Diameter at Small End	d_{ai}	6.106 in	1.697 in
Root Diameter at End	d_{fe}	7.927 in	1.709 in
Vertex Distance	A_e	0.879 in	3.970 in
Vertex Distance at Small End	A_i	0.666 in	3.007 in
Pitch Cone Angle	δ	75.9638 deg	14.0362 deg
Outside Cone Angle	δ_a	77.7003 deg	15.7551 deg
Root Cone Angle	δ_f	73.8802 deg	11.9527 deg
Facewidth	b	1.000 in	
Facewidth Ratio	b_r	0.2425 ul	
Addendum	a^*	1.0000 ul	1.0000 ul
Clearance	c^*	0.2000 ul	0.2000 ul
Root Fillet	r_f^*	0.3000 ul	0.3000 ul
Whole Depth of Tooth	h_e	0.275 in	0.274 in
Tooth Thickness at End	s_e	0.196 in	0.196 in
Chordal Thickness	t_c	0.173 in	0.173 in
Chordal Addendum	a_c	0.093 in	0.092 in
Limit Deviation of Helix Angle	F_β	0.00051 in	0.00043 in
Limit Circumferential Run-out	F_r	0.00110 in	0.00067 in
Limit Deviation of Axial Pitch	f_{pt}	0.00035 in	0.00030 in
Limit Deviation of Basic Pitch	f_{pb}	0.00033 in	0.00028 in
Equivalent Number of Teeth	z_v	263.879 ul	16.492 ul
Equivalent Pitch Diameter	d_v	28.985 in	1.812 in
Equivalent Outside Diameter	d_{va}	29.205 in	2.029 in
Equivalent Base Circle Diameter	d_{vb}	27.237 in	1.702 in
Unit Correction without Tapering	x_z	-6.9273 ul	0.5525 ul
Unit Correction without Undercut	x_p	-14.4314 ul	0.0380 ul
Unit Correction Allowed Undercut	x_d	-14.5985 ul	-0.1291 ul
Addendum Truncation	k	0.0000 ul	0.0102 ul
Unit Outside Tooth Thickness	s_a	0.8288 ul	0.6824 ul

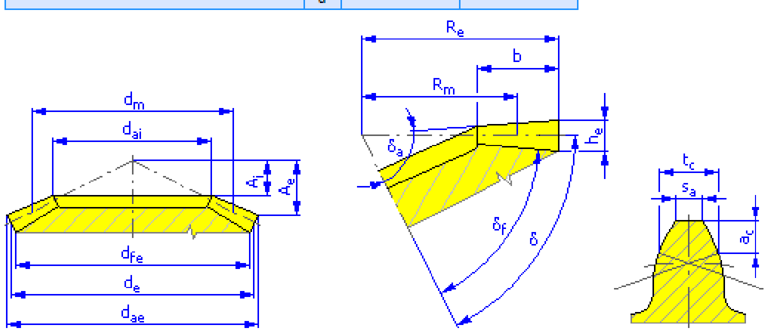


Table 9: ANSI/AGMA 2001-D04 calculations for ring and pinion gear

⊟ Strength Calculation

⊟ Factors of Additional Load

Overload Factor	K_o	1.200 ul	
Dynamic Factor	K_v	1.035 ul	
Size Factor	K_s	1.000 ul	1.000 ul
Reliability Factor	K_R	1.000 ul	
Temperature Factor	k_t	1.000 ul	
Load Distribution Factor	K_m	1.173 ul	1.173 ul
Lead Correction Factor	C_{mc}	1.000 ul	1.000 ul
Mesh Alignment Correction Factor	C_e	1.000 ul	
Pinion Proportion Modifier	C_{pm}	1.000 ul	
Mesh Alignment Factor	C_{ma}	Commercial Enclosed Gear Units (0.1427)	

⊟ Factors for Contact

Surface Condition Factor	C_f	1.000 ul	1.000 ul
Stress Cycle Factor	Z_N	1.472 ul	1.402 ul
Hardness Ratio Factor	C_H	1.000 ul	1.000 ul
Elastic Factor	C_p	268.326 ul	
Geometry Factor	I	0.117 ul	

⊟ Factors for Bending

Reverse Loading Factor	Y_a	1.000 ul	1.000 ul
Rim Thickness Factor	K_B	1.000 ul	1.000 ul
Stress Cycle Factor	Y_N	1.731 ul	1.545 ul
Geometry Factor	J	0.300 ul	0.300 ul

⊟ Results

Factor of Safety from Pitting	k_f	3.644 ul	3.469 ul
Factor of Safety from Tooth Breakage	k_n	12.010 ul	10.720 ul
Check Calculation		**Positive**	

Table 10: Common parameters for the spider gears

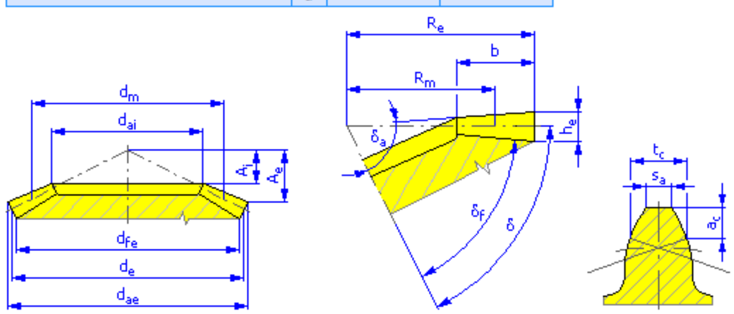
⊟ Common Parameters

Gear Ratio	i	1.0000 ul
Tangential Diametral Pitch	P_{et}	9.000 ul/in
Helix Angle	β	0.0000 deg
Tangential Pressure Angle	α_t	20.0000 deg
Shaft Angle	Σ	90.0000 deg
Normal Pressure Angle at End	α_{ne}	20.0000 deg
Contact Ratio	ε	1.6712 ul
Limit Deviation of Axis Parallelity	f_x	0.00043 in
Limit Deviation of Axis Parallelity	f_y	0.00022 in
Virtual Gear Ratio	i_v	1.000 ul
Equivalent Center Distance	a_v	2.894 in
Virtual Center Distance	a_n	2.894 in
Pitch Cone Radius	R_e	1.807 in
Pitch Cone Radius in Middle Plane	R_m	1.447 in

Table 11: Spider gear dimensions

⊟ Gears

		Gear 1	Gear 2
Type of model		Component	Component
Number of Teeth	z	23 ul	23 ul
Unit Correction	x	0.0000 ul	-0.0000 ul
Tangential Displacement	x_t	0.0000 ul	-0.0000 ul
Pitch Diameter at End	d_e	2.556 in	2.556 in
Pitch Diameter in Middle Plane	d_m	2.046 in	2.046 in
Outside Diameter at End	d_{ae}	2.713 in	2.713 in
Outside Diameter at Small End	d_{ai}	1.631 in	1.631 in
Root Diameter at End	d_{fe}	2.367 in	2.367 in
Vertex Distance	A_e	1.199 in	1.199 in
Vertex Distance at Small End	A_i	0.721 in	0.721 in
Pitch Cone Angle	δ	45.0000 deg	45.0000 deg
Outside Cone Angle	δ_a	48.5185 deg	48.5185 deg
Root Cone Angle	δ_f	40.7801 deg	40.7801 deg
Facewidth	b	0.720 in	
Facewidth Ratio	b_r	0.3987 ul	
Addendum	a^*	1.0000 ul	1.0000 ul
Clearance	c^*	0.2000 ul	0.2000 ul
Root Fillet	r_f^*	0.3000 ul	0.3000 ul
Whole Depth of Tooth	h_e	0.244 in	0.244 in
Tooth Thickness at End	s_e	0.175 in	0.175 in
Chordal Thickness	t_c	0.154 in	0.154 in
Chordal Addendum	a_c	0.083 in	0.083 in
Limit Deviation of Helix Angle	F_β	0.00043 in	0.00043 in
Limit Circumferential Run-out	F_r	0.00083 in	0.00083 in
Limit Deviation of Axial Pitch	f_{pt}	0.00033 in	0.00033 in
Limit Deviation of Basic Pitch	f_{pb}	0.00031 in	0.00031 in
Equivalent Number of Teeth	z_v	32.527 ul	32.527 ul
Equivalent Pitch Diameter	d_v	2.894 in	2.894 in
Equivalent Outside Diameter	d_{va}	3.072 in	3.072 in
Equivalent Base Circle Diameter	d_{vb}	2.719 in	2.719 in
Unit Correction without Tapering	x_z	0.0524 ul	0.0524 ul
Unit Correction without Undercut	x_p	-0.8999 ul	-0.8999 ul
Unit Correction Allowed Undercut	x_d	-1.0670 ul	-1.0670 ul
Addendum Truncation	k	0.0000 ul	0.0000 ul
Unit Outside Tooth Thickness	s_a	0.7445 ul	0.7445 ul



⊟ Loads

		Gear 1	Gear 2
Power	P	0.100 hp	0.098 hp
Speed	n	100.00 rpm	100.00 rpm
Torque	T	5.252 lbforce ft	5.147 lbforce ft
Efficiency	η	0.980 ul	
Tangential Force	F_t	61.605 lbforce	
Normal Force	F_n	65.559 lbforce	
Radial Force (direction 1)	F_{r1}	15.855 lbforce	15.855 lbforce
Radial Force (direction 2)	F_{r2}	15.855 lbforce	15.855 lbforce
Axial Force (direction 1)	F_{a1}	15.855 lbforce	15.855 lbforce
Axial Force (direction 2)	F_{a2}	15.855 lbforce	15.855 lbforce
Circumferential Speed	v	0.893 fps	
Resonance Speed	n_{E1}	27707.136 rpm	

⊟ Material

		Gear 1	Gear 2
		User material	User material
Ultimate Tensile Strength	S_u	85300 psi	101500 psi
Yield Strength	S_y	45500 psi	49300 psi
Modulus of Elasticity	E	380000 psi	380000 psi
Poisson's Ratio	μ	0.400 ul	0.400 ul
Bending Fatigue Limit	σ_{Flim}	52200.0 psi	51100.0 psi
Contact Fatigue Limit	σ_{Hlim}	60900.0 psi	165300.0 psi
Hardness in Tooth Core	JHV	210 ul	210 ul
Hardness in Tooth Side	VHV	600 ul	600 ul
Base Number of Load Cycles in Bending	N_{Flim}	3000000 ul	3000000 ul
Base Number of Load Cycles in Contact	N_{Hlim}	50000000 ul	100000000 ul
W?hler Curve Exponent for Bending	q_F	6.0 ul	6.0 ul
W?hler Curve Exponent for Contact	q_H	10.0 ul	10.0 ul
Type of Treatment	type	0 ul	2 ul

Table 12: ANSI/AGMA 2004-D04 spider gear calculations

⊟ Strength Calculation

⊟ Factors of Additional Load

Application Factor	K_A	1.200 ul	
Dynamic Factor	K_{Hv}	1.007 ul	1.007 ul
Face Load Factor	$K_{H\beta}$	1.012 ul	1.009 ul
Transverse Load Factor	$K_{H\alpha}$	1.000 ul	1.000 ul
One-time Overloading Factor	K_{AS}	1.000 ul	

⊟ Factors for Contact

Elasticity Factor	Z_E	22.280 ul	
Zone Factor	Z_H	2.495 ul	
Contact Ratio Factor	Z_ε	0.881 ul	
Bevel Gear Factor	Z_k	0.850 ul	
Single Pair Tooth Contact Factor	Z_B	1.004 ul	1.004 ul
Life Factor	Z_N	1.600 ul	1.600 ul
Lubricant Factor	Z_L	0.937 ul	
Roughness Factor	Z_R	1.000 ul	
Speed Factor	Z_v	0.878 ul	
Helix Angle Factor	Z_β	1.000 ul	
Size Factor	Z_X	1.000 ul	1.000 ul

⊟ Factors for Bending

Form Factor	Y_{Fa}	2.499 ul	2.499 ul
Stress Correction Factor	Y_{Sa}	1.702 ul	1.702 ul
Teeth with Grinding Notches Factor	Y_{Sag}	1.000 ul	1.000 ul
Helix Angle Factor	Y_β	1.000 ul	
Contact Ratio Factor	Y_ε	0.699 ul	
Bevel Gear Factor	Y_k	1.000 ul	
Alternating Load Factor	Y_A	1.000 ul	1.000 ul
Production Technology Factor	Y_T	1.000 ul	1.000 ul
Life Factor	Y_N	2.500 ul	2.500 ul
Notch Sensitivity Factor	Y_δ	1.226 ul	1.210 ul
Size Factor	Y_X	1.000 ul	1.000 ul
Tooth Root Surface Factor	Y_R	1.000 ul	

⊟ Results

Factor of Safety from Pitting	S_H	15.755 ul	42.763 ul
Factor of Safety from Tooth Breakage	S_F	45.909 ul	44.391 ul
Static Safety in Contact	S_{Hst}	25.039 ul	27.130 ul
Static Safety in Bending	S_{Fst}	93.653 ul	91.679 ul
Check Calculation		**Positive**	

Sample Calculation

Hertzian contact analysis for cylindrical contact.

$$b = \sqrt{\frac{2F}{\pi l} \cdot \frac{\frac{(1-\upsilon_1^2)}{E_1} + \frac{(1}{}}{\frac{1}{d_1} + \frac{1}{d_2}}} \tag{1}$$

$$b = \sqrt{\frac{2 \cdot 100}{\pi \cdot 0.5906} \cdot \frac{\frac{(1-0.4^2)}{326.3} + \frac{(1-0.4^2)}{326.3}}{\frac{1}{0.173} + \frac{1}{0.173}}}$$

$$2b = 0.0014$$

$$P_{max} = \frac{2F}{\pi \cdot b \cdot l} \tag{2}$$

$$P_{max} = \frac{2F}{\pi \cdot b \cdot l}$$

$$P_{max} = \frac{2 \cdot 100 lb_f}{\pi \cdot 0.007 \cdot 0.5922}$$

$$P_{max} = 15537.3\ PSI$$

$$\sigma_x = 2\nu P_{max}\left[\sqrt{\left(1 + \frac{z^2}{b^2}\right)} - \left|\frac{z}{b}\right|\right] \tag{3}$$

$$\sigma_y = -P_{max}\left[\frac{1 + 2 \cdot \frac{z^2}{b^2}}{\sqrt{\left(1 + \frac{z^2}{b^2}\right)}} - 2\left|\frac{z}{b}\right|\right] \tag{4}$$

$$\sigma_z = \frac{-P_{max}}{\sqrt{\left(1 + \frac{z^2}{b^2}\right)}} \tag{5}$$

$$\tau_{xz} = \frac{\sigma_x - \sigma_z}{2} \tag{6}$$

$$\tau_{yz} = \frac{\sigma_x - \sigma_z}{2} \tag{7}$$

Equations 3-7 can be used to make a graph and interpolate maximum shear stress. (Figure 31)

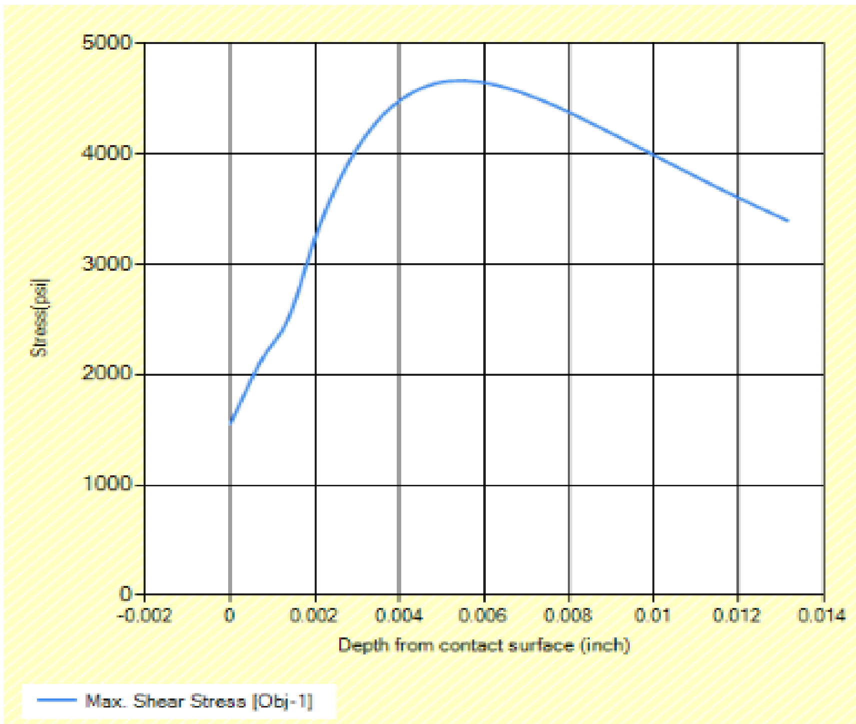


Figure 31. Graph of Stress Versus Depth of Contact Surface.

Find the maximum point on this graph results in a maximum shear stress of 4665.6 PSI at a depth of 0.005 inches

All further calculations were performed in Inventor. Inventor can perform calculations from a variety of standards. The standard in which all calculations were performed was ANSI/AGMA 2001-D04. These calculations are for determining the pitting resistance and bending strength of helical involute gears.

Safety factor of contact fatigue

$$S_{H1,2} = \frac{s_{ac} \cdot Z_{N1,2} \cdot C_{H1,2}}{C_p \cdot K_T \cdot K_R \cdot \sqrt{F_t \cdot K_o \cdot K_v \cdot K_{s1,2} \cdot \frac{K_{m1,2}}{d_{m1} \cdot b_w} \frac{C_{f1,2}}{I}}}$$

Figure 32. Safety factor of contact fatigue formula

Safety factor of bending fatigue

$$S_{F1,2} = \frac{s_{at1,2} \cdot Y_{N1,2} \cdot Y_{A1,2}}{K_T \cdot K_R \cdot F_t \cdot K_o \cdot K_v \cdot K_{s1,2} \cdot \frac{P_t}{b_{wF1,2}} \frac{K_{m1,2} \cdot K_{B1,2}}{J_{1,2}}}$$

Figure 33. Safety factor of bending fatigue formula

Discussion and Analysis

The calculated values obtained from the Hertzian contact calculations are not comparable to the calculations performed by Inventor, as Inventor measures different parameters of the system. Inventor bases the calculations from ANSI/AGMA 2001-D04 which calculates the safety factor of contact fatigue, and the safety factor of bending fatigue. These two parameters are important as they define the lifetime of the gear sets. While the Hertzian calculations will only provide a graph plotting the maximum stress a tooth can withstand. The Hertzian calculation provided a maximum stress of 15500 PSI that the gear teeth can withstand. The gears will never experience those types of stresses for the team's use case. The gears were not the main issue when designing the differential. The geometric tolerances were an unforeseen issue as we are inexperienced with designing assemblies like this. The spider gears that were mounted onto axles would prove to be difficult. The lack of geometric tolerance caused the shafts to not be perfectly aligned with the carrier, causing the shafts to deflect due to the irregular gear contact. While deflection is an issue the differential did still function and was able to spin the shafts at different speeds. Another issue was using plywood as a case material. Plywood being an amorphous material proved difficult to use fasteners with. This is what prompted the decision to use nails and dowels, as it would minimize any splitting or splintering of the wood. Even with using those fasteners the team still had issues with the plywood splitting even when nailing the small finishing nails. Observing other parts of the assembly, the 3D printed parts proved to be the best part about the assembly, as their easy manufacturing and precise tolerances were paramount to manufacturing quality gears.

Conclusions and Recommendations

We were able to construct a small-scale functional model of a rear car differential using rudimentary materials with surprisingly tight tolerances. We created the shaft by utilizing the lathe and the milling machine, created inventor drawings of the gears and utilized the 3D printer to print them, and we created a wooden casing by cutting and assembling plywood plates. The final product was able to perform its function at 0.1 horsepower. A recommendation for repeating the process would be the use of a material different than plywood for the casings and supports. 3D printing the pieces would remove the error from cutting holes for the shafts and would allow for the ability to integrate supports for the shafts into the outer walls to prevent the shafts from dipping. Another recommendation would be to increase the hole sizes for the loose fits in the Inventor drawings to allow for more room and less chance for shaft contact. These changes would allow for a smaller, tighter casing with added supports and would remove some excess friction caused by poor gear contact due to shafts dipping.

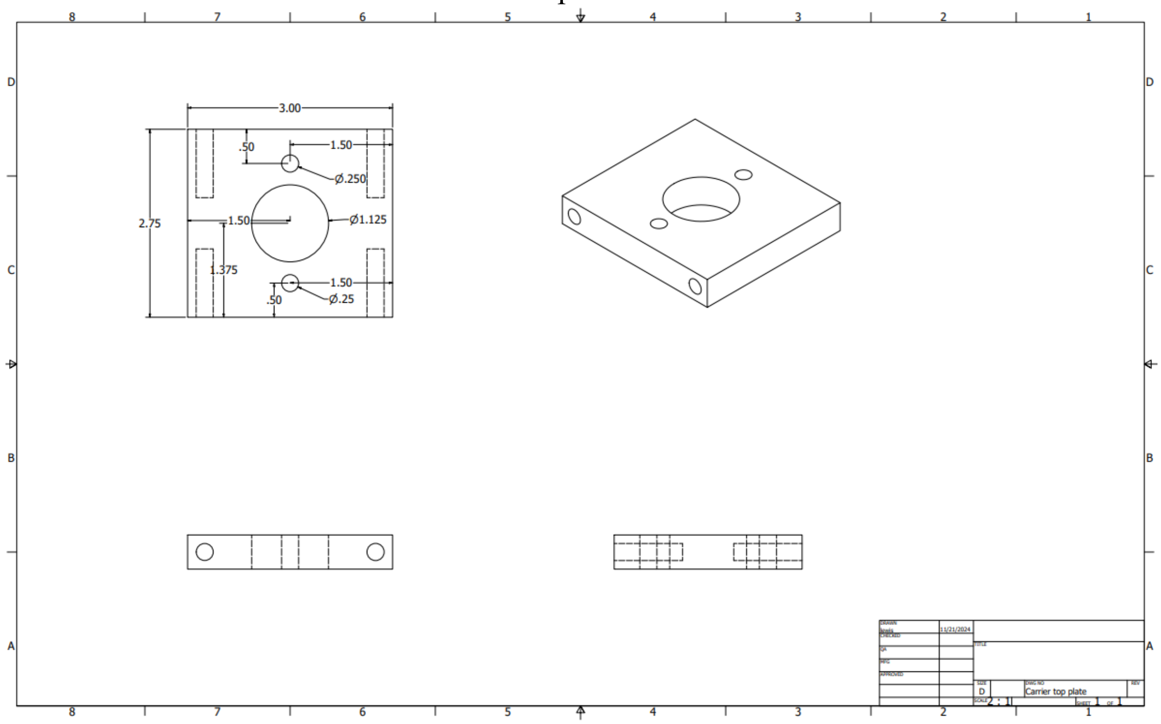
References

Kalpakjian, Serope, and Steven R. Schmid. *Manufacturing Engineering and Technology*. Pearson, 2020.

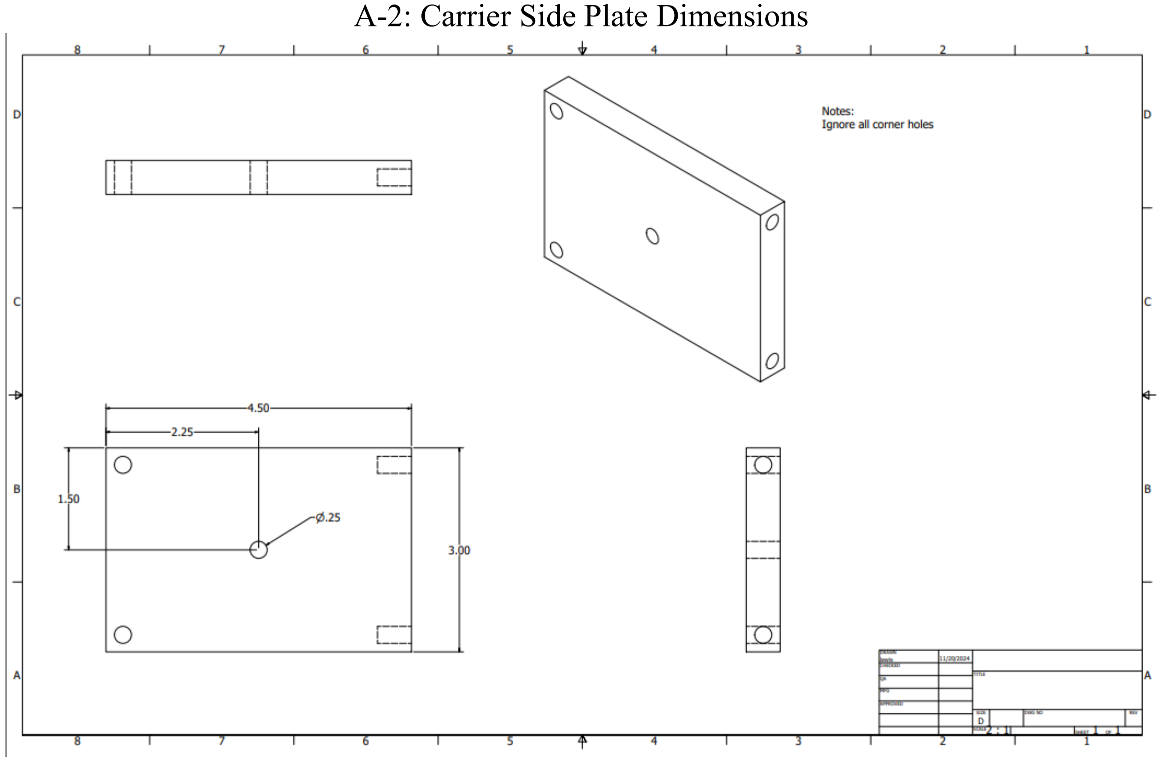
American Gear Manufacturers Association. (2004). *AGMA 2001-D04: Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth.* Alexandria, VA: AGMA.

Appendix

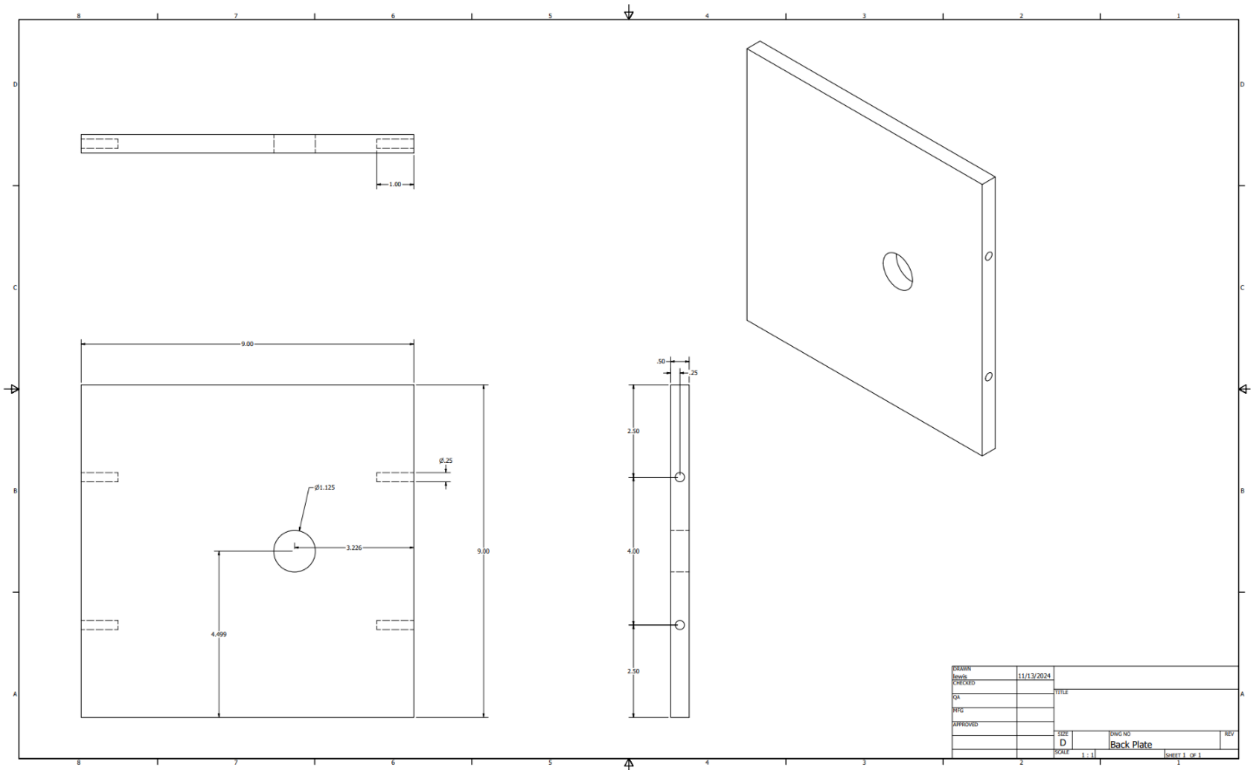
A-1: Carrier Top Plate Dimensions



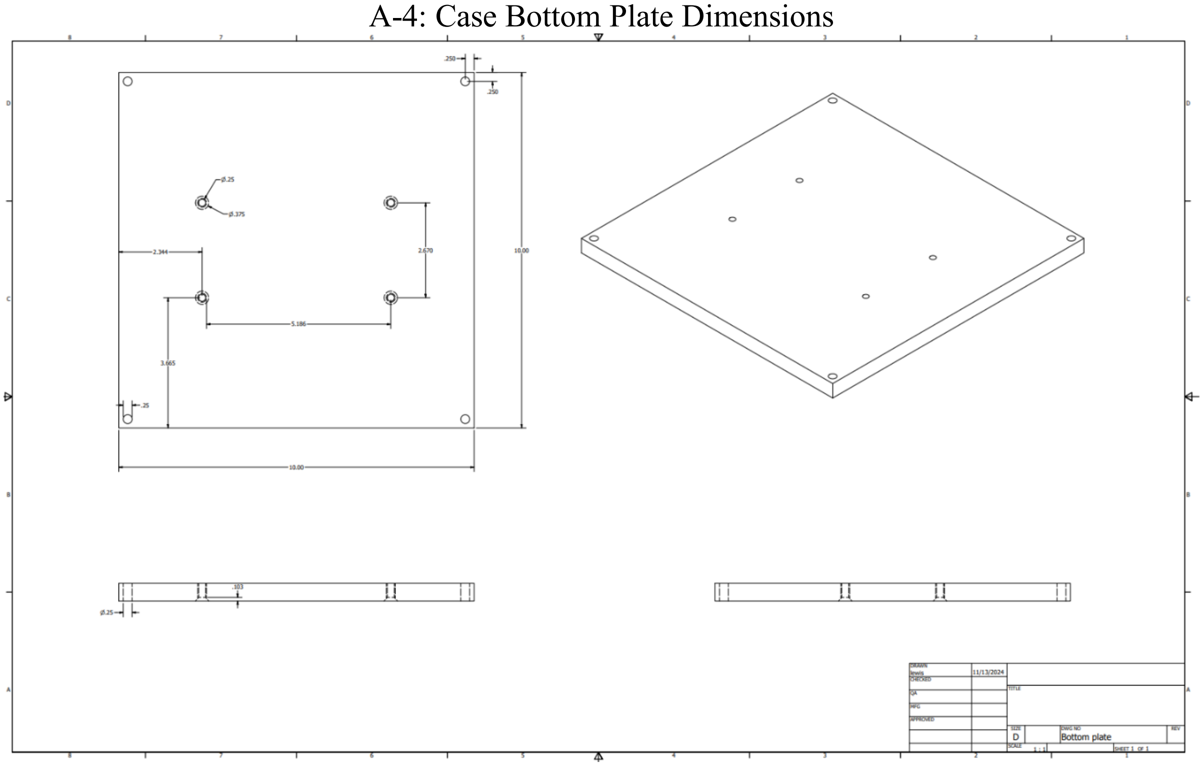
A-2: Carrier Side Plate Dimensions



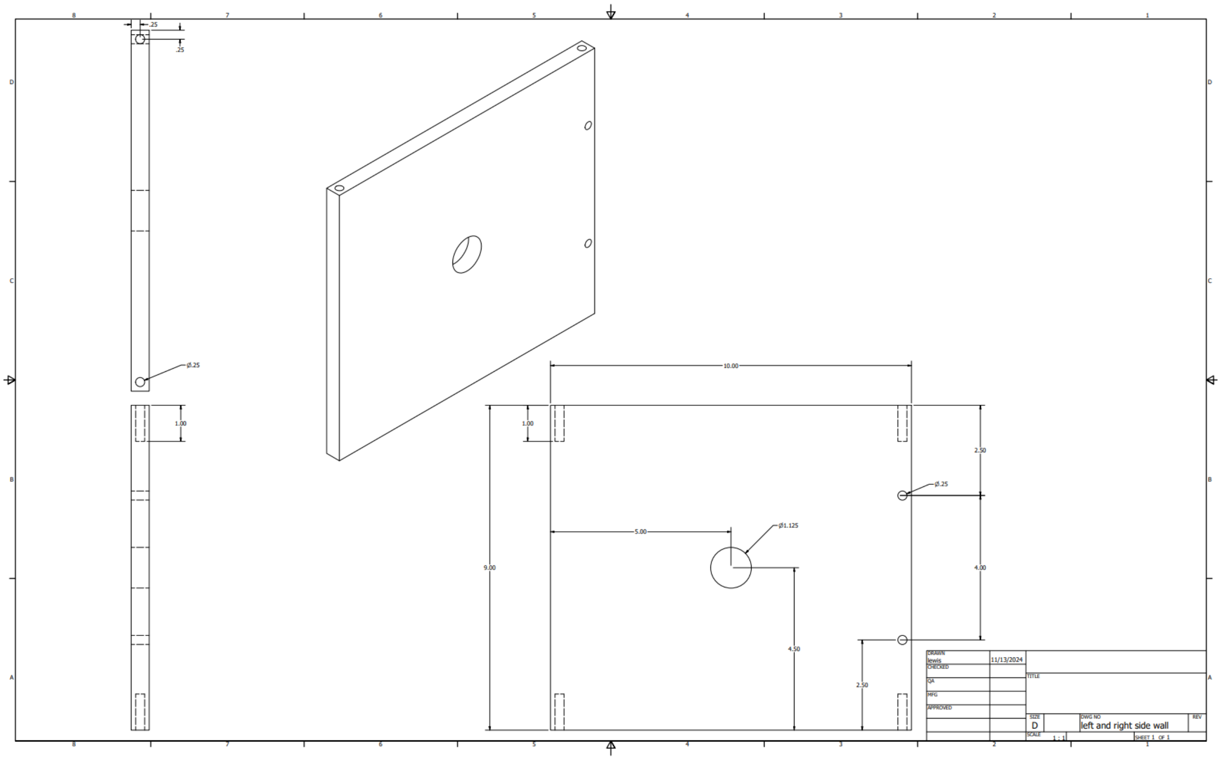
A-3:Case Back Plate Dimensions



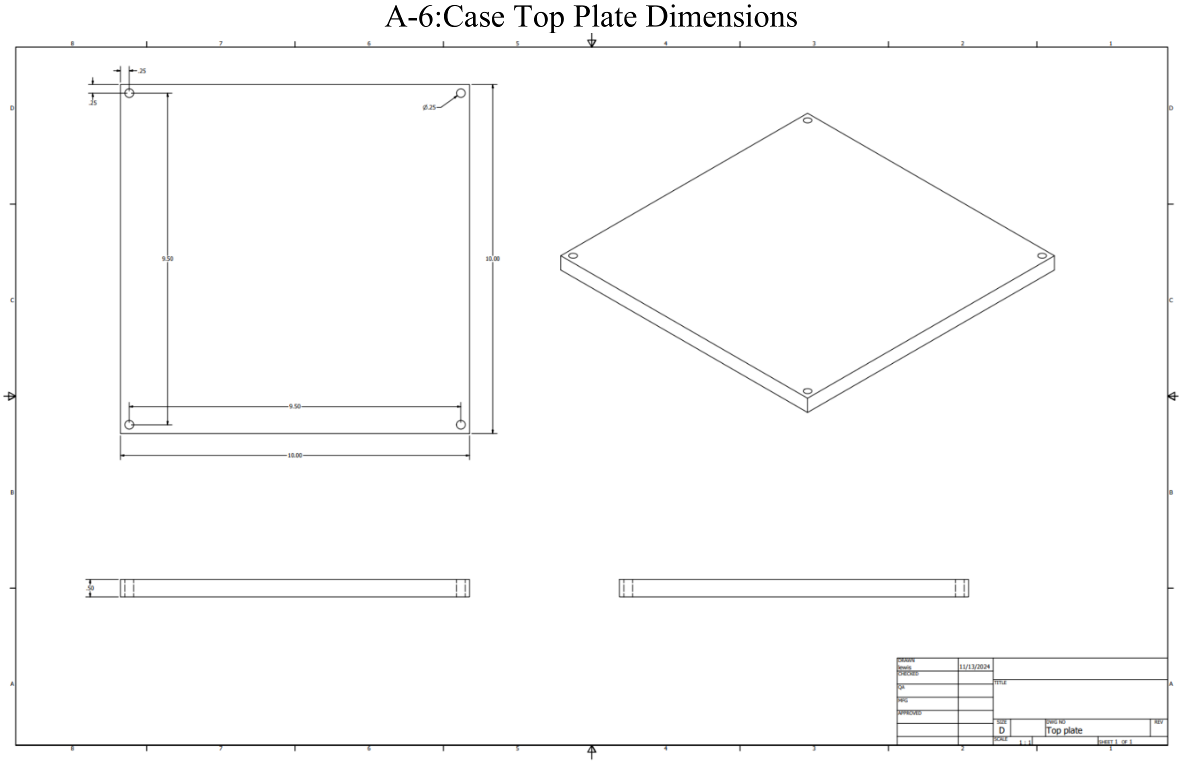
A-4: Case Bottom Plate Dimensions



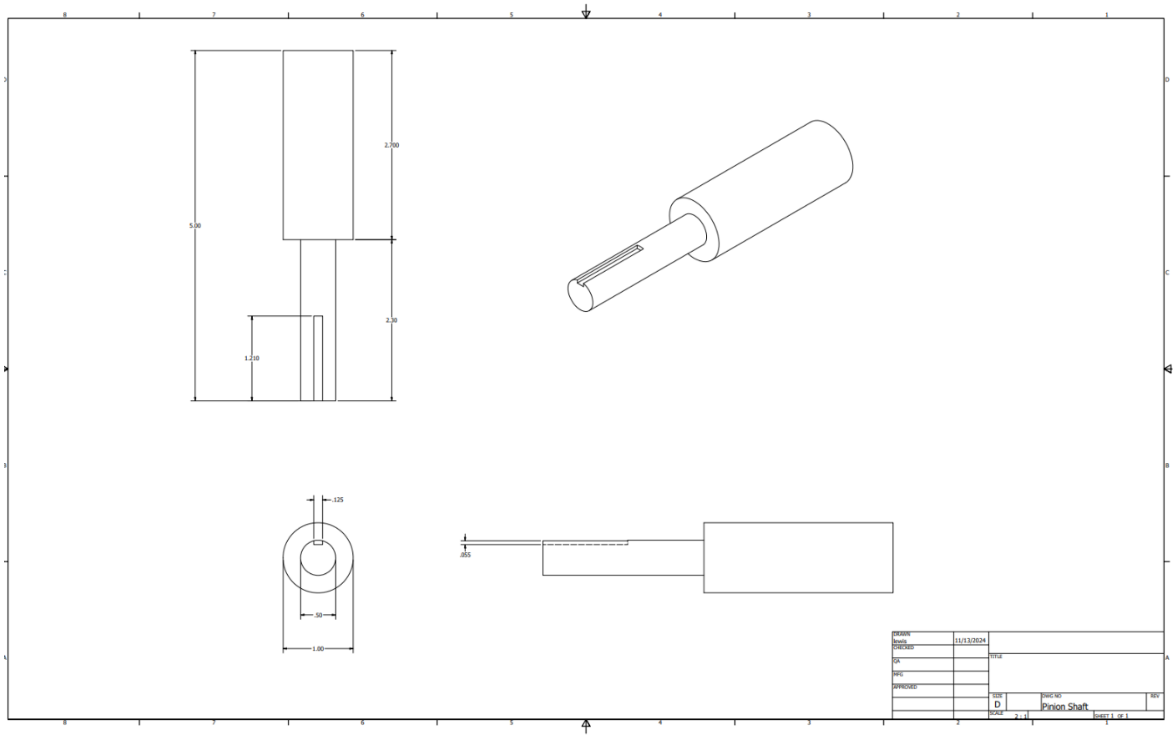
A-5: Case Side Plate Dimensions



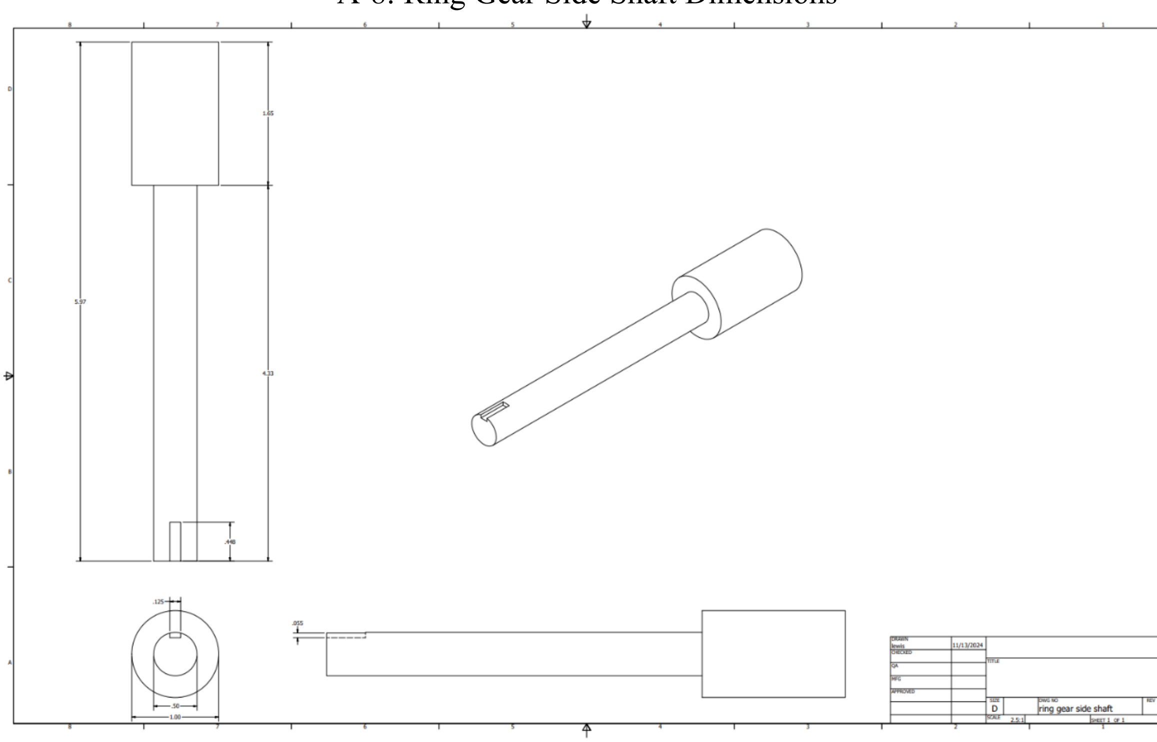
A-6:Case Top Plate Dimensions



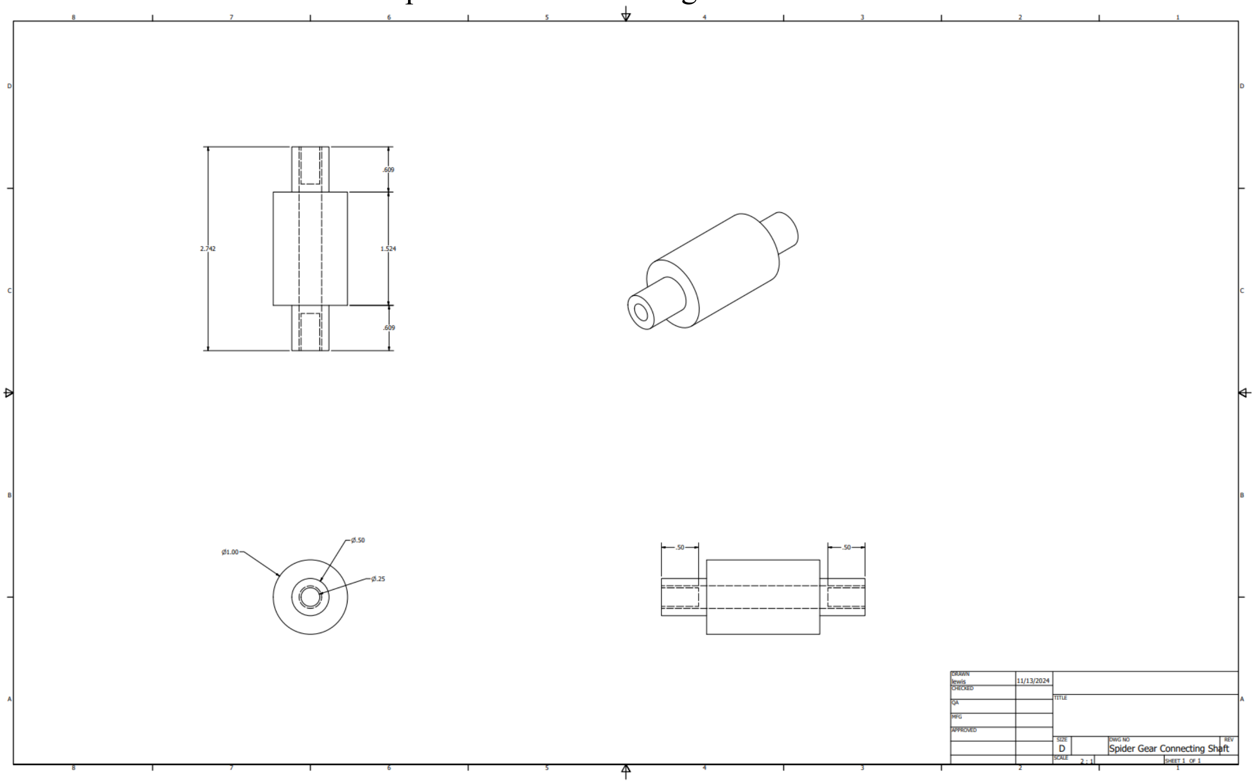
A-7: Pinion Shaft Dimensions



A-8: Ring Gear Side Shaft Dimensions



A-9: Spider Gear Connecting Shaft Dimensions



A-10: Spider Gear Side Shaft Dimensions